NTK/KW/15 – 5839

Third Semester B. Sc. Examination

STATISTICS

Paper – I

(Statistical Methods)

Time : Three Hours] [Max. Marks : 50

N. B. : All questions are compulsory and carry equal marks.

1. (A) Let the joint probability function of discrete random variables x and y be given by

$$f(x, y) = \begin{cases} \dfrac{x + 2y}{27} & ,\ x = 0, 1, 2 \\ & \ \ y = 0, 1, 2 \\ 0 & ,\ \text{elsewhere} \end{cases}$$

(i) Present the joint distribution of x and y in a bivariate table.

(ii) Find the marginal distribution of random variables x and y.

(iii) Find the mean and variance of random variable x.

(iv) Derive the conditional distribution of random variable y given x = 2.

(v) Calculate E(y| x = 2). 10

OR

(E) Suppose the conditional distribution of X given y is

$$f_{x/y}\,(x/y) = \frac{e^{-y}\, y^{x}}{x_{!}}\ ,\ x = 0,\ 1,\ 2\ ---$$

The random variable Y is continuous with marginal p.d.f.

$$f_y(y) = e^{-y}\ ,\ y = \geq 0$$

Find the marginal p.d.f of X.

(F) If x and y are two continuous random variables with joint p.d.f

$$f(x, y) = \begin{cases} \frac{3}{5}\, x(x + y)\ ,\ 0 < x < 1,\ 0 < y < 2 \\ 0,\ \text{elsewhere.} \end{cases}$$

Find :

(i) $P\left(0 < x < \frac{1}{2}\ ,\ 0 < y < 2\right)$

(ii) Marginal density if X

(iii) Variance of X.

(iv) Conditional density of y given X = x.

5 + 5

2. (A) State the p.d.f of Bivariate normal distribution of a pair of random variables X and Y. Find m.g.f of it and hence find correlation coefficient between X and Y. 10

OR

(E) If random variables X_1 and X_2 follow trinomial distribution, state its p.m.f. Find its m.g.f. and hence find mean and variance of X_1. Also find covariance between X_1 and X_2.

According to Mendelian theory of heredity, if plants with round yellow seeds are crossbred with plants with wrinkled green seeds, the probabilities of getting a plant that produces round yellow seeds, wrinkled yellow seeds and round green seeds are respectively, $\frac{9}{16}, \frac{4}{16}, \frac{3}{16}$. What is the probability that among 9 plants three obtained true will be 4 round yellow seeds, three wrinkled yellow seeds and three round green seeds ?

10

3. (A) If X_1 and X_2 are independent standard normal variables, find the joint p.d.f. $Y_1 = X_1 - X_2$ and $Y_2 = X_1 + X_2$. Are Y_1 and Y_2 independent. How are Y_1 and Y_2 distributed ?

(B) If the joint probability distribution of X and Y given by

$$f(x, y) = \begin{cases} \frac{(x-y)^2}{7}, & x = 1, 2 ; y = 1, 2, 3 \\ 0, & \text{elsewhere} \end{cases}$$

(i) Write the bivariate distribution of X and y in tabular form.

(ii) If U = x + y and V = x – y then write bivariate probability distribution table of U and V.

5 + 5

OR

(E) Let X_1 and X_2 be independent random variables following same exponential distribution with parameter θ. Find m.g.f. of X_1 and use it to show that :

(i) $Y = X_1 + X_2$

(ii) $Z = \frac{X_1 + X_2}{2}$

follow Gamma distribution. Identify the parameters.

(F) Let there be two fair four sided dice–die 1 with face numbers 0, 1, 2, 3 and die 2 with face numbers 0, 4, 8 12.

Let X and Y be the outcomes on die 1 and die 2 respectively when these are rolled.

Derive :

(i) The p.m. f of X and p.m.f. of Y.

(ii) The joint p.m.f of X and Y.

(iii) Let w = x + y. Derive the joint distribution of w and x.

(iv) Show that w follows discrete uniform distribution. 5 + 5

4. (A) Define Chisquare variable with parameter n. State its p.d.f. find m.g.f. of it and hence find mean and variance. State and prove additive property of Chisquare distribution. 10

OR

(E) Define F variable. State its p.d.f. Find mean and mode of F distribution.

(F) Derive distribution of

(i) Sum of n independent Poisson variables.

(ii) Linear combination of n independent normal variables. 5 + 5

5. Solve any **Ten** questions from the following :—

(A) Let x and y have the joint p.m.f given by

y \ x	1	2	3	4
1	0.1	0.2	0.3	0.05
2	0.05	0.05	0.15	0.1

Find E(Y).

(B) Show that correlation coefficient of two independent randum variables X and Y is zero. 5 + 5

(C) The joint p.d.f of x and y is given by

$$f(x,\ y) = \begin{cases} \frac{3}{2}\, y^2 , & 0 \leq x \leq 2,\ 0 \leq y \leq 1 \\ 0 , & \text{elsewhere} \end{cases}$$

Check whether X and Y are stochastically independent.

(D) If the joint probability mass function of random variables X and Y given by

$$f(x, y) = \frac{2!}{x!\ y!\ (2 - x - y)!} \left(\frac{1}{4}\right)^{x} \left(\frac{2}{4}\right)^{y} \left(\frac{1}{4}\right)^{2-x-y}$$

for $0 \leq x + y \leq 2$ where x and y are non negative integers. State the probability mass functions of X and Y.

(E) If a pair of random variables X and Y follows Bivariate normal distribution with parameters $\mu_x = 10$, $\mu_y = 12$, $6_x^2 = 9$ $6_y^2 = 16$, $\varrho = 0.6$, Find E(y/x=5).

(F) If a pair of random variables x and y follows Bivariate normal distribution, write expression for V(y/x).

(G) What is sampling distribution ?

(H) State R command for drawing a random sample of size 20 from normal population with mean 50 and standard deviation 3.

(I) If $X_1 \sim B(n_1, p)$ and $X_2 \sim B(n_2, p)$. What is the distribution of $X_1 + X_2$?

(J) Show that students's t can be regarded as a particular case of Fisher's t.

(K) State the relationship between variables t and F.

(L) Show that t distribution is symmetric.

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